

# Resonances Required: Dynamical Analysis of the 24 Sex and HD 200964 Planetary Systems

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## ABSTRACT

We perform several suites of highly detailed dynamical simulations to investigate the architectures of the 24Sextantis and HD 200964 planetary systems. The best fit orbital solution for the two planets in the 24Sex system places them on orbits with periods that lie very close to 2:1 commensurability, while that for the HD 200964 system places the two planets therein in orbits whose periods lie close to a 4:3 commensurability. In both cases, the proposed best-fit orbits are mutually crossing - a scenario that is only dynamically feasible if the planets are protected from close encounters by the effects of mutual mean motion resonance. Our simulations reveal that the best fit orbits for both systems lie within narrow islands of dynamical stability, and are surrounded by much larger regions of extreme instability. As such, we show that the planets are only feasible if they are currently trapped in mutual mean-motion resonance - the 2:1 resonance in the case of 24Sex b and c, and the 4:3 resonance in the case of HD 200964 b and c. In both cases, the region of stability is strongest and most pronounced when the planetary orbits are mutually coplanar. As the inclination of planet c with respect to planet b is increased, the stability of both systems rapidly collapses.

*Subject headings:* planetary systems – methods: numerical – planets and satellites: individual (24 Sex b, 24 Sex c, HD 200964b, HD 200964c – stars: individual (24 Sextantis, HD 200964

## 1. Introduction

Systems of multiple extrasolar planets provide rich laboratories for testing theories of planet formation and dynamical evolution. Radial-velocity planet-search programs, with observational baselines approaching 20 years, often discover that apparent single-planet systems

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show evidence for additional planets. These discoveries are due to a combination of increased time coverage and ongoing improvements in measurement precision arising from innovation and refinement in instrumentation, observing strategies, and analysis techniques (Vogt et al. 2010; Mayor et al. 2011). The former enables the detection of Jupiter-analog planets with orbital periods  $P \gtrsim 10$  yr (Wright et al. 2009; Jones et al. 2010; Wittenmyer et al. 2012), and the latter permits the robust detection of planets of ever lower mass. Wright et al. (2009) showed that at least 28% of planetary systems surveyed by Doppler programs host multiple planets. Recent results from the *Kepler* spacecraft, with some 885 multiply-transiting planet candidates (Fabrycky et al. 2012), have vastly increased the pool of these fascinating and valuable planetary systems.

A number of recent studies have highlighted the need for observational detections of multiple-planet systems to be supported by dynamical investigations that test whether the orbits of the proposed planets are dynamically feasible. These investigations can reveal that the planetary system configuration as reported is catastrophically unstable (Horner et al. 2011; Wittenmyer et al. 2012a; Hinse et al. 2012a; Goździewski et al. 2012; Horner et al. 2012). Alternately, detailed dynamical simulations of systems close to mutual mean-motion resonance (MMR) can provide important additional constraints on the parameters of the planets (e.g. Horner et al. (2012); Robertson et al. (2012a)). This is critical because any commensurability between the orbital periods of two planets can result in either extreme stability or instability, depending on the precise orbits of the planets involved. For example, Robertson et al. (2012a) presented revised orbits for the two Jovian planets in the HD 155358 system (Cochran et al. 2007), potentially placing them in mutual 2:1 MMR. The dynamical stability analysis of Robertson et al. (2012a) showed that, for the orbital architecture presented in that work, the 2:1 MMR was indeed a region of long-term stability, serving as further evidence that the new orbital parameters are correct. In the HD 204313 three-planet system (Ségransan et al. 2010), a similar dynamical analysis reveals that the observed two outermost Jovian planets must be trapped in mutual 3:2 MMR (Robertson et al. 2012), an unusual architecture which dynamical mapping showed to be an island of extreme stability in the system. In that case, the dynamical analysis was critically necessary to constrain the HD 204313 system architecture, demonstrating that the two gas giant planets are locked in the 3:2 MMR.

The intriguing results of these detailed stability investigations have prompted us to take a close look at other planetary systems which appear to be in or near low-order MMRs. In recent years, a number of surveys (Hatzes et al. 2005; Sato et al. 2005; Johnson et al. 2006b; Döllinger et al. 2007; Johnson et al. 2011b; Wittenmyer et al. 2011b) have discovered a significant number of planetary systems orbiting intermediate-mass stars ( $M_* > 1.5M_\odot$ ). These stars have proven to be a fertile hunting ground for interesting planetary systems.

Of particular interest are the two low-order resonant giant planet pairs in the 24 Sex (2:1) and HD 200964 (4:3) systems (Johnson et al. 2011a). For both systems, the best-fit radial-velocity solutions result in crossing orbits, an architecture which can only be dynamically stable on long timescales if the planets are protected from mutual close encounters by resonant motion.

In their study of 24 Sex and HD 200964, Johnson et al. (2011a) performed small-scale  $n$ -body dynamical simulations which showed, as expected, that the long-term stable solutions were restricted to the 2:1 (24 Sex) and 4:3 (HD 200964) mean-motion resonances. The authors noted that their simulation results could not yet conclusively confirm that the planets were in resonance, and urged further, more detailed dynamical investigation.

Resonant protection, such as that proposed for the 24 Sex and HD 200964 systems, is well known and studied in our Solar system. A wealth of Solar system objects move on orbits that would be highly unstable, were it not for the influence of such resonances, including the Hilda asteroids (Franklin et al. 1993), Jovian and Neptunian Trojans (Morbidelli et al. 2005; Sheppard & Trujillo 2006; Lykawka & Horner 2010; Horner et al. 2012) and, most famously, the dwarf planet Pluto and its brethren, the Plutinos (Malhotra 1995; Friedland 2001). Taken in concert with the growing catalogue of resonant exoplanets, these populations highlight the important role resonant dynamics plays in the formation and evolution of planetary systems. Indeed, astronomers studying the formation and evolution of our Solar system have learned a great deal about the extent, pace, and nature of the migration of the giant planets through studies of the system’s resonant small body populations (e.g. Malhotra (1995); Morbidelli et al. (2005); Lykawka et al. (2009); Brož et al. (2011)), revealing that the formation of our Solar system was most likely a relatively chaotic process.

In this paper, we apply our highly detailed dynamical analysis techniques to the extremely interesting 24 Sex and HD 200964 planetary systems. In Section 2, we briefly describe the simulation methods and initial conditions. Section 3 gives the results, and we give our conclusions in Section 4.

## 2. Numerical Methods

To study the dynamics of the two planetary systems proposed in Johnson et al. (2011a), we performed two main suites of dynamical simulations - one for each of the planetary systems studied. We followed the strategy we have successfully employed to study the dynamics of a number of other exoplanetary systems (e.g. Marshall et al. (2010); Horner et al. (2011); Wittenmyer et al. (2012a); Horner et al. (2012)), and followed the dynamical evolution of a

large number of different architectures for each system using the Hybrid integrator within the  $n$ -body dynamics package MERCURY (Chambers 1999). In each case, we placed the better constrained of the two planets in question (24 Sex b and HD 200964 b) on its nominal best-fit orbit at the start of our integrations. The orbital parameters of the planets simulated in this work are shown in Table 1<sup>1</sup>. For the other planet (24 Sex c and HD 200964 c), we then tested 41 unique orbital semi-major axes, distributed uniformly across the full  $\pm 3\sigma$  range allowed by the uncertainties in that planet’s orbit. For each of these 41 possible semi-major axes, we tested 41 unique eccentricities, again spread evenly across the full  $\pm 3\sigma$  range of allowed values. For each of these 1681  $a - e$  values, we tested 15 values of the longitude of the planet’s periastron,  $\omega$ , and 5 values of its mean anomaly,  $M$ , each spanning the appropriate  $\pm 3\sigma$  error ranges. In this way, a total of 126,075 potential architectures were tested for each system.

In each of our simulations, we followed the evolution of the two planets involved for a period of up to 100 Myr, until they were either ejected from their system, collided with one another, or were thrown into their central star. The times at which collisions and ejections occurred were recorded, which allowed us to create dynamical maps of the system’s stability (Figures 1 and 3).

In addition to the main suites of integrations discussed above, we performed five additional suites of integrations for each of the two planetary systems to investigate the influence that the mutual inclinations of the planetary orbits would have on their stability. In this, we followed Wittenmyer et al. (2012a), and considered cases where planet c was initially moving on an orbit inclined by 5, 15, 45, 135 and 180 degrees with respect to that of planet b. Due to the significant computational overhead in performing such runs, the resolution of these subsidiary investigations was lower than for the main runs. For each of these additional runs, a total of 11,025 trials were carried out, yielding the results shown in Figures 2 and 4.

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<sup>1</sup>The orbital solutions for the planets were taken from the Exoplanets Data Explorer website, <http://exoplanets.org>, on the 27th March 2012. We note that there are very small differences between the elements presented in that explorer and those presented in Johnson et al. (2011a). These are, however, far smaller than the  $1\sigma$  errors on the values, and have no effect on our results or conclusions

### 3. Results and Discussion

#### 3.1. The 24 Sextantis System

Figure 1 shows the results of our dynamical simulations for the 24 Sex 2:1 resonant system. At each point in the  $(a, e)$  grid, the small colored cell represents the mean survival time of 75 variants of the two-planet system – each with a unique initial combination of longitude of periastron and mean anomaly for the outer planet. The initial orbital parameters for the inner planet were held fixed, as noted in the previous section. The best-fit orbit for the outer planet is shown as an open box with crosshairs indicating the  $1\sigma$  uncertainties in semi-major axis and eccentricity. The orbital solution of Johnson et al. (2011a) lies directly on the narrow region of stable orbits, with mean survival times  $> 10^6$  yr. This is strong evidence that the planets are truly in 2:1 resonance – that the resonance is *required* for the stability of the system. Nearly all of the surrounding parameter space is highly unstable, with survival times typically less than  $10^4$  yr. An additional small region of stability can be seen centered at  $a \sim 2.2$  AU, for eccentricities below around 0.3. That region is the result of an overlapping web of weak, high-order resonances that congregate in the region 2.174 - 2.243 AU<sup>2</sup>. Interestingly, exactly the same feature can be seen in Figure 9 of Robertson et al. (2012a). The HD 155358 planetary system is somewhat analogous to that around 24 Sex, in that it features two planets that are most likely trapped in mutual 2:1 MMR. In that case, HD 155358c is most likely moving on an orbit that is somewhat less eccentric than that proposed for 24 Sex c, and so their Figure 9 shows the dynamical stability of that system to lower eccentricities. As the eccentricity of the outermost planet falls, the broad region of stability offered by these overlapped high-order resonances broadens until it merges with that offered by the protection of the 2:1 MMR. We note, however, that the stability region for 24 Sex c offered by those higher order resonances lies well away from the central  $\pm 1\sigma$  of the allowed orbital architectures for the system. Hence it seems far more reasonable to conclude that the planets in this system are, most likely, trapped in mutual 2:1 MMR.

If the two planets were scattered to their present locations by a distant body, or by mutual chaotic interactions during their migration (Barnes et al. 2011; Marzari & Weidenschilling 2002), it is possible that they have some non-zero inclination relative to each other. To ex-

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<sup>2</sup>More explicitly, the following commensurabilities are found in that region - 31:15 at 2.174 AU; 29:14 at 2.177 AU; 27:13 at 2.181 AU; 25:12 at 2.186 AU; 23:11 at 2.191 AU; 21:10 at 2.197 AU; 19:9 at 2.205 AU; 17:8 at 2.215 AU; 15:7 at 2.227 AU; 28:13 at 2.235 AU; 13:6 at 2.244 AU. At low eccentricities, such resonances can help to ensure the stability of orbits that would otherwise be somewhat unstable, as has been observed for both Solar system objects and exoplanetary systems (e.g. Lykawka et al. (2011); Robertson et al. (2012a); Deck et al. (2012)).

plore the effect of mutually-inclined scenarios, we performed a subsidiary suite of integrations for a range of mutual inclination angles: 5, 15, 45, 135, and 180 degrees (i.e. coplanar but retrograde). These runs were set up as previously described, except at lower resolution: each scenario consists of a grid of 21 values of  $a$ , 21 of  $e$ , 5 of  $\omega$ , and 5 of  $M$  (11025 total trial systems). The results are given in Figure 2, and show that the system becomes generally more unstable when the planets depart from a prograde, coplanar configuration. Notably, the two retrograde scenarios had dramatically shorter lifetimes (panels e and f). We note that while the retrograde coplanar case (panel f) shows a long-term stable region in the lower right, that region is quite far from the  $1\sigma$  uncertainty on the orbit, much like our previous work on the proposed HU Aquarii planetary system (Wittenmyer et al. 2012a).

### 3.2. The HD 200964 System

Figure 3 shows the results for the HD 200964 4:3 resonant system. As in Figure 1, the open box with crosshairs shows the best-fit parameters for the outer planet (Johnson et al. 2011a). Once again, we see that the radial-velocity solution for this system, with the planets in a 4:3 resonance, lies within a narrow region of orbital stability surrounded by highly unstable parameter space. As was the case for the 24 Sex system, this dynamical map shows that the resonance is required for the system’s stability. An interesting feature of the 4:3 resonant protection is that the region of stability does not extend all the way to zero eccentricity - in other words, some small, non-zero eccentricity is required for HD 200964c to be dynamically stable within the 4:3 MMR with HD 200964b. This instability at very low eccentricities is observed in the Solar system’s Plutino population (trapped in 3:2 MMR with Neptune; see e.g. Fig. 6 of Robertson et al. (2012)), and was also observed in some of the more extreme integrations of the planetary system orbiting HD 142 (Wittenmyer et al. 2012).

At the far right-hand edge of the allowed range, for  $a > 2.05$  AU, we see a highly stable region at all tested eccentricities. This feature is a common outcome of dynamical stability results at moderate and low eccentricities, located just interior to the location of the 2:1 MMR. A similar feature can be seen in Fig 1 of Horner et al. (2011), for the otherwise dynamically unfeasible HU Aquarii planetary system. It typically represents the inner edge of the region for which dynamically stable solutions become the norm, rather than the exception, apart from resonant interactions. At greater separations, this region of stability extends to ever greater eccentricities, since the boundary between stable and unstable solutions is determined, for the non-resonant case, by the closest approach distance between the two planets in question. In Horner et al. (2011), this inner edge was discussed in terms

of the Hill Radius,  $R_H$ , of the more massive, innermost planet proposed in that work. The divide between stable and unstable orbits was found to follow a line that (roughly) followed a line of constant periastron distance for the outermost planet, centred on a periastron distance between 3 and 5 Hill radii beyond the orbit of the innermost planet. For objects on near-circular orbits, the Hill Radius can be approximated as

$$R_H = a \sqrt[3]{\frac{m}{3M}}$$

where  $a$  and  $m$  are the semi-major axis and mass of the planet in question, and  $M$  is the mass of the central star. For HD 200964, the sharp boundary between unstable and stable orbits at around 2.05 AU once again lies between three and four Hill radii beyond the orbit of the innermost planet. Interestingly, we note that Chambers et al. (1996) find that a system of two planets on low eccentricity, low inclination orbits “is stable with respect to close encounters if the initial semi-major axis difference,  $\Delta$ , measured in mutual Hill radii,  $R_H$ , exceeds  $2\sqrt{3}$ , due to conservation of energy and angular momentum.” The mutual Hill radius,  $R_{HM}$  is defined as

$$R_{HM} = \sqrt[3]{\left(\frac{m_b+m_c}{3M}\right)\left(\frac{a_b+a_c}{2}\right)}$$

where  $m_b$ ,  $m_c$ ,  $a_b$ , and  $a_c$  are the masses and semi-major axes of planets b and c, and  $M$  is the mass of the central star. Given the criterion that the inner edge of the stable region should be found when the planets are separated by a distance of  $2\sqrt{3}$  times their mutual Hill radius, it is therefore trivial to work out where the inner edge of this stable region should be expected to lie, for the case where the orbits are circular. Holding the location of planet b fixed, we thus find that the inner edge of the stable region should lie at  $a = 2.15\text{AU}$ , a little more distant than that observed in our Figure 3. Once again, however, it should be noted that this stable region lies well beyond the central  $\pm 1\sigma$  region, and so represents a significantly less likely architecture for the HD 200964 system.

As for the 24 Sex system, we also considered mutually-inclined scenarios, running a further series of simulations as described in Section 3.1. The results for the HD 200964 system are the same: the mean lifetime decreases significantly when the planets are inclined by more than 90 degrees with respect to each other (retrograde orbits). Again, the most stable configuration was prograde and coplanar (panel a, identical to Figure 3).

#### 4. Conclusions

Both the 24 Sex and HD 200964 systems host giant planets in close resonances (Johnson et al. 2011a). We have performed detailed dynamical simulations testing the  $3\sigma$  range of allowed

parameter space for these two systems. Our results have further constrained the orbital parameters, with the best-fit solutions falling directly in narrow ( $\sim 1\sigma$  width) strips of long-term stability. We also find that the stability of both systems is strongly dependent on the mutual inclinations of the planets involved, with coplanar orbits offering by far the greatest potential for dynamically stable solutions to be found. This work demonstrates the utility of such dynamical mapping for better understanding the architectures of multiple-planet systems. The results of this work confirm that the resonant configurations are indeed required for long-term stability in the 24 Sex and HD 200964 systems. This adds them to a very short list of low-order-resonant exoplanetary systems, which are extremely valuable test cases for understanding giant-planet formation and migration processes.

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Table 1. Planetary System Parameters

Planet	Period (days)	$T_0$ (JD-2400000)	$e$	$\omega$ (degrees)	K ( $\text{m s}^{-1}$ )	$M \sin i$ ( $\text{M}_{\text{Jup}}$ )	$a$ (AU)
24 Sex b	$455.2 \pm 3.2$	$54758 \pm 30$	$0.184 \pm 0.029$	$227 \pm 20$	$33.2 \pm 1.6$	$1.6 \pm 0.2$	$1.338 \pm 0.024$
24 Sex c	$910 \pm 21$	$54941 \pm 30$	$0.412 \pm 0.064$	$172 \pm 9$	$23.5 \pm 2.9$	$1.4 \pm 0.2$	$2.123 \pm 0.049$
HD 200964 b	$613.8^{+1.3}_{-1.4}$	$54916 \pm 30$	$0.040^{+0.04}_{-0.02}$	$288^{+47}_{-112}$	$34.5^{+2.7}_{-1.5}$	$1.84^{+0.14}_{-0.08}$	$1.601 \pm 0.002$
HD 200964 c	$825.0^{+5.1}_{-3.1}$	$55029 \pm 130$	$0.181^{+0.024}_{-0.058}$	$182.6^{+67.7}_{-57.1}$	$15.4 \pm 3.2$	$0.895^{+0.123}_{-0.063}$	$1.944 \pm 0.041$

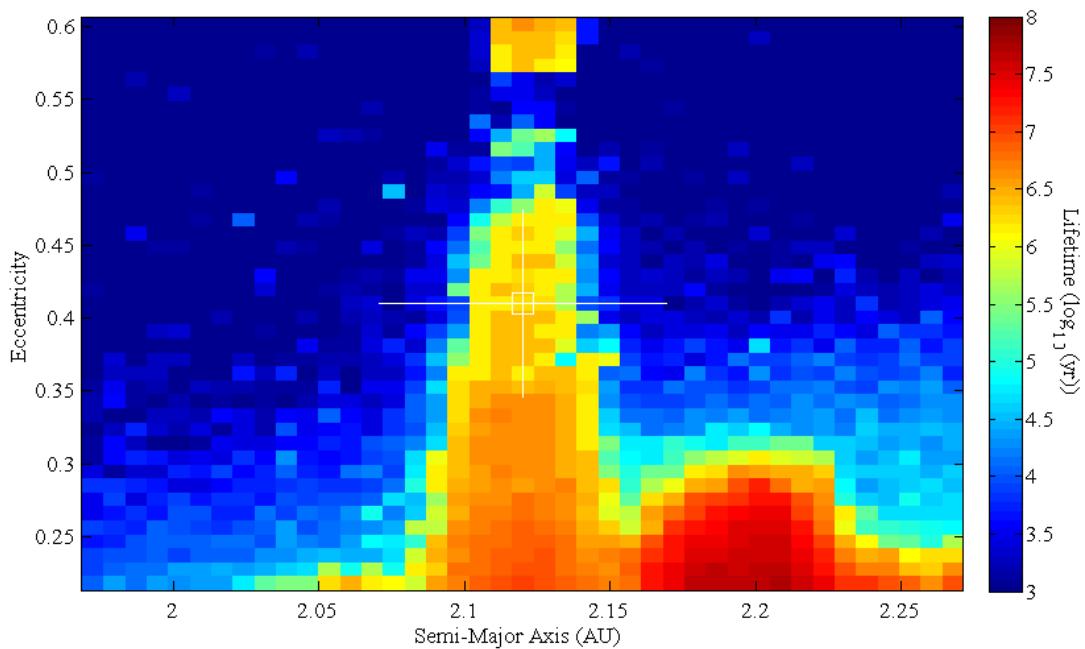


Fig. 1.— Dynamical stability for the 24 Sex system as a function of the initial semimajor axis and eccentricity of the outer planet. The nominal best-fit orbit for that planet is marked by the open square, and the  $1\sigma$  uncertainties are shown by the crosshairs. The 2:1 resonance appears as a narrow strip of stability which coincides with the outer planet’s best-fit orbit (Johnson et al. 2011a).

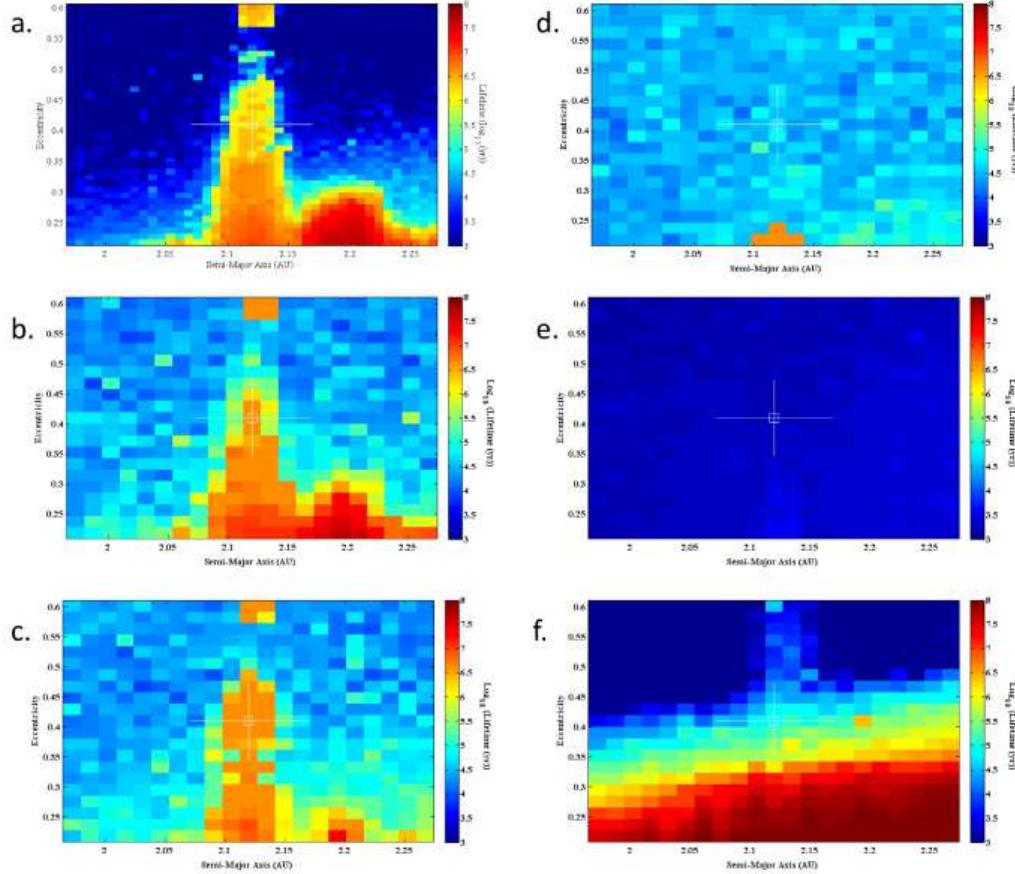


Fig. 2.— Dynamical stability for the 24 Sex system, but for six values of inclination between the two planets. Panels (a) through (f) represent mutual inclinations of 0, 5, 15, 45, 135, and 180 degrees, respectively. Panel (a) is a duplicate of Figure 1, shown here for ease of comparison. As in previous figures, the color bar represents the log of the mean survival time.

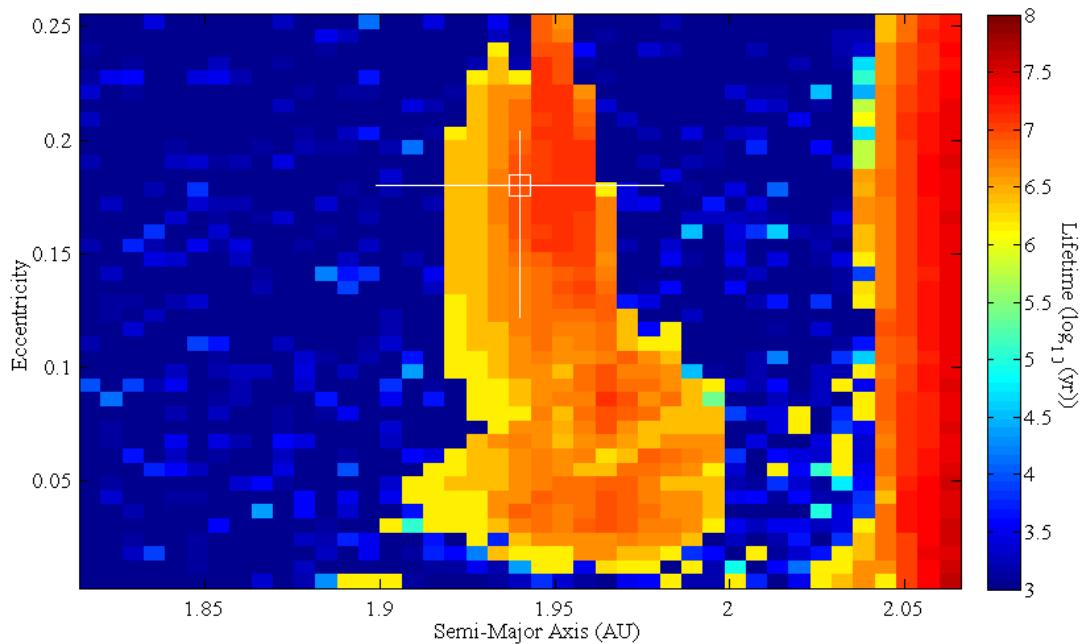


Fig. 3.— Dynamical stability for the HD 200964 system as a function of the initial semimajor axis and eccentricity of the outer planet. The nominal best-fit orbit for that planet is marked by the open square, and the  $1\sigma$  uncertainties are shown by the crosshairs. The 4:3 resonance appears as a narrow strip of stability which coincides with the outer planet’s best-fit orbit (Johnson et al. 2011a).

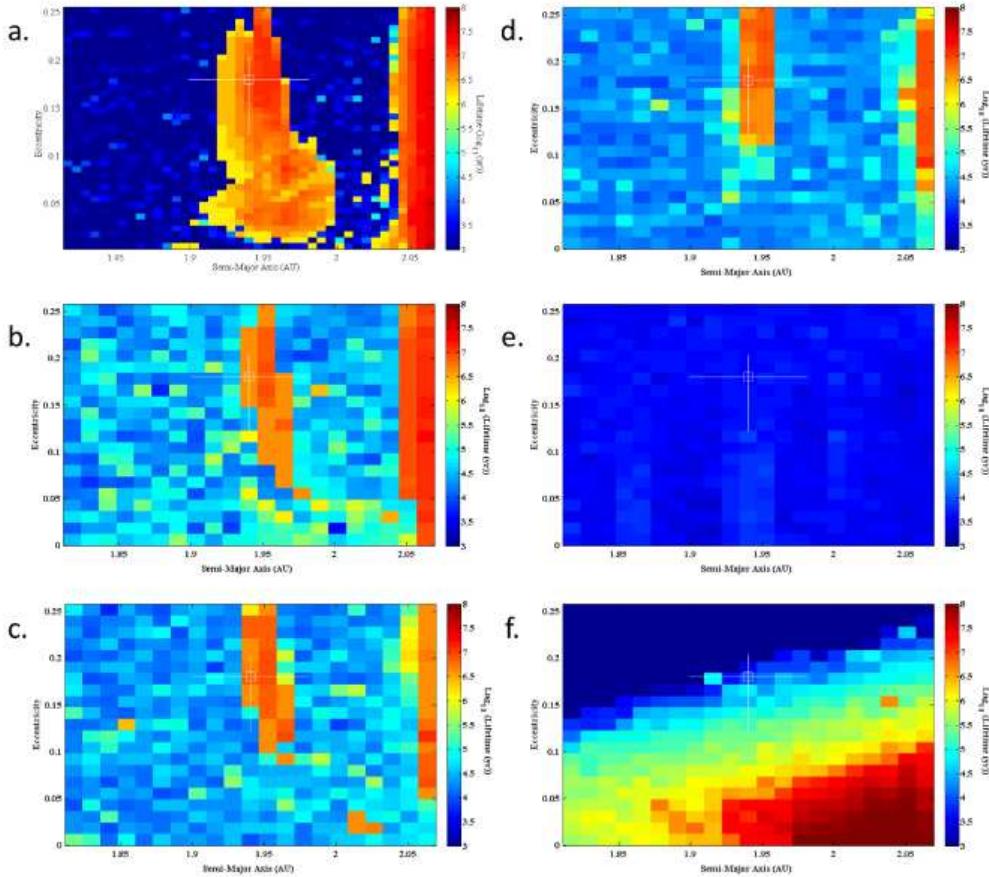


Fig. 4.— Same as Figure 2, but for the HD 200964 system. As for 24 Sex, mutual-inclination scenarios are much less stable than the prograde-coplanar scenario in panel (a).